The astronauts in the upper left of this photo are working on the space shuttle. As they orbit the Earth—at a rather high speed—they experience apparent weightlessness. The Moon, in the background, also is orbiting the Earth at high speed. Both the Moon and the space shuttle move in nearly circular orbits, and each undergoes a centripetal acceleration. What keeps the Moon and the space shuttle (and its astronauts) from moving off in a straight line away from Earth? It is the force of gravity. Newton's law of universal gravitation states that all objects attract all other objects with a force proportional to their masses and inversely proportional to the square of the distance between them.

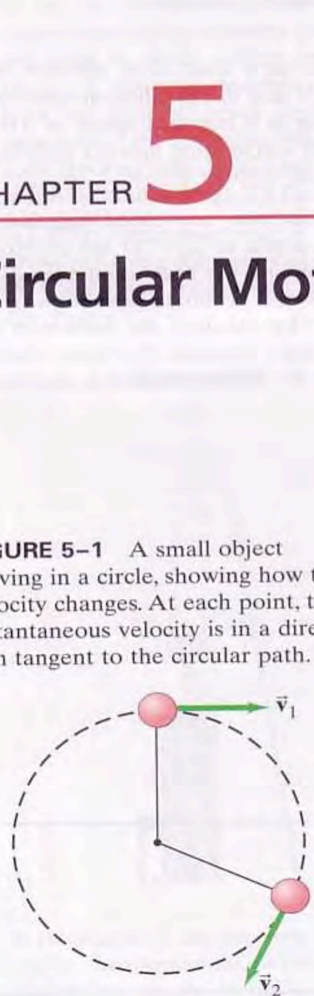
CHAPTER 5

Circular Motion; Gravitation

An object moves in a straight line if the net force on it acts in the direction of motion, or the net force is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string revolving around one's head, or the nearly circular motion of the Moon about the Earth.

In this Chapter, we study the circular motion of objects, and how Newton's laws of motion apply. We also discuss how Newton conceived of another great law by applying the concepts of circular motion to the motion of the Moon and the planets. This is the law of universal gravitation, which was the capstone of Newton's analysis of the physical world.

FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.



5–1 Kinematics of Uniform Circular Motion

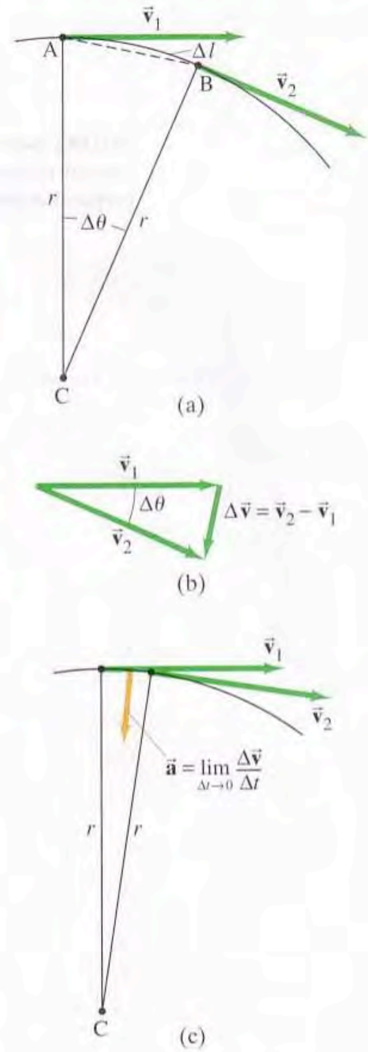
An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity continuously changes as the object moves around the circle (Fig. 5–1). Because acceleration is defined as the rate of change of velocity, a change in direction of velocity constitutes an acceleration, just as a change in magnitude of velocity does. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant $(v_1 = v_2 = v)$. We now investigate this acceleration quantitatively.

Acceleration is defined as

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t},$$

where $\Delta\vec{\mathbf{v}}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation in which Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing, Fig. 5–2, we consider a nonzero time interval. During the time interval Δt, the particle in Fig. 5–2a moves from point A to point B, covering a distance Δl *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \Delta\vec{\mathbf{v}}$, and is shown in Fig. 5–2b.

FIGURE 5–2 Determining the change in velocity, $\Delta\vec{\mathbf{v}}$, for a particle moving in a circle. The length Δl is the distance along the arc, from A to B.



If we let Δt be very small (approaching zero), then Δl and $\Delta\theta$ are also very small, and $\vec{\mathbf{v}}_2$ will be almost parallel to $\vec{\mathbf{v}}_1$; $\Delta\vec{\mathbf{v}}$ will be essentially perpendicular to them (Fig. 5–2c). Thus $\Delta\vec{\mathbf{v}}$ points toward the center of the circle. Since $\vec{\mathbf{a}}$, by definition, is in the same direction as $\Delta\vec{\mathbf{v}}$, it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-pointing" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\vec{\mathbf{a}}_R$.

We next determine the magnitude of the centripetal (radial) acceleration, a_R. Because CA in Fig. 5–2a is perpendicular to $\vec{\mathbf{v}}_1$, and CB is perpendicular to $\vec{\mathbf{v}}_2$, it follows that the angle $\Delta\theta$, defined as the angle between CA and CB, is also the angle between $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$. Hence the vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, and $\Delta\vec{\mathbf{v}}$ in Fig. 5–2b form a triangle that is geometrically similar[†] to triangle CAB in Fig. 5–2a. If we take $\Delta\theta$ to be very small (letting Δt be very small) and setting $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change, we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta l}{r}.$$

This is an exact equality when Δt approaches zero, for then the arc length Δl equals the cord length AB. We want to find the instantaneous acceleration, so we let Δt approach zero, write the above expression as an equality, and then solve for Δv:

$$\Delta v = \frac{v}{r}\,\Delta l.$$

To get the centripetal acceleration, a_R, we divide Δv by Δt:

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta l}{\Delta t}.$$

But $\Delta l/\Delta t$ is just the linear speed, v, of the object, so

$$a_R = \frac{v^2}{r}. \qquad \textbf{(5–1)}$$

Centripetal (radial) acceleration

Equation 5–1 is valid even when v is not constant.

To summarize, *an object moving in a circle of radius r at constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is* $a_R = v^2/r$. It is not surprising that this acceleration depends on v and r. The greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

CAUTION

In uniform circular motion, the speed is constant, but the acceleration is not zero

[†] Appendix A contains a review of geometry.