Binding Energy Background

Since all of the protons in a nucleus carry a positive charge, we can imagine that they genuinely detest each other and are actively striving to flee from each other's presence. (I've been to some meetings like that...) Anyway, a substantial expense is required to hold them there, and it's called **binding energy.** The amount of binding energy divided by the number of nucleons gives the "cost" of those nucleons joining that particular nuclear "club." This binding energy per nucleon is a good index to the stability or strength of a nucleus.

To find **binding energy per nucleon** here's what you do:

1) Find Z, the number of protons, and N, the number of neutrons in the nucleus.

> Example: Consider Na-23, the ordinary sodium in salt and in our bodies.
> All sodium nuclei have 11 protons (see Serway Appendix H), so
> sodium-23 must have 12 neutrons, since $N = A-Z$.

2) Calculate the total mass of the parts of the atom, recalling that each proton has a mass of **1.007 825 u**, while each neutron has a mass of **1.008 665 u**.

So, the total mass of the parts is:

$$11\ (1.007\ 825\ \text{u}) + 12\ (1.008\ 665\ \text{u}) = 11.086\ 075\ \text{u} + 12.10\ 398\ \text{u} = 23.190\ 055\ \text{u}$$

3) Look up (column 6) and subtract the actual mass of the atom:

$$23.190\ 055\ \text{u} - 22.989\ 767\ \text{u} = 0.200\ 288\ \text{u}$$

4) Calculate the energy equivalent to this mass, using $E = mc^2$, where in this unit system **$c^2 = 931.5$ MeV/u**.

$(0.200\ 288\ \text{u})\ (931.5\ \text{MeV/u}) = 186.568\ 272\ \text{MeV}$. Rounding to four digits, $E = 186.6$ MeV. That's the *total* binding energy of sodium-23.

5) Finally, divide the total binding energy by the number of nucleons yielding the **binding energy *per* nucleon:**

186.6 MeV / 23 = 8.11 MeV (typically rounded to two decimal places)

Binding Energy Practice

1) Look up the actual mass of carbon-12. It is *defined* as the value you see in the table. What, then, is the definition of the atomic mass unit, **u**?

2) Calculate the binding energy per nucleon of the other two isotopes of sodium, Na-22 and Na-24, both of which are radioactive.

What do you find about their energies, compared with that of Na-23? Based on that scanty evidence alone, what can you guess about the connection between binding energy per nucleon and nuclear stability?

3) Take the date of the day of the month when you were born and multiply it by 3. Calculate the binding energy per nucleon in the nuclide that has Z equal to your number. If there are several isotopes, choose the one that is most abundant, or that has the longest half-life. (For example, if you were born on the 7th of June, so you would multiply 7 times 3 to get 21. Element 21 is Scandium and its only isotope in the Serway Appendix B is Sc-45, so I would find its binding energy per nucleon.)

4) Add your result to the class data table and use the data table to construct a graph of binding energy per nucleon vs. atomic number. Let's do this as a class using Logger Pro on the board. We'll scale the horizontal axis from 0-95 or 100 since that's about the number of different elements and scale the vertical axis from 5 MeV to 9 MeV. (Note: We are going forego including the origin on this graph since there are no interesting binding energies/nucleon that small.)

Now let’s answer some questions about the Binding Energy/Nucleon graph you made.

5) What element appears to have the most tightly bound nucleus (largest binding energy per nucleon)?

Elements with fewer protons than this can release energy via nuclear fusion and this is what powers stars and allows them to make elements smaller than this. But what about elements with larger numbers of protons? Those can release energy by nuclear fission (think Uranium or Plutonium).

5) If stars can't normally make the heaviest elements like gold and uranium, how *are* they produced? The answer lies in one of the most powerful explosions in the universe—a supernova. Supernova can synthesize heavy elements like gold and lead because of their tremendous energy release. A number of reactions can take place, but foremost are nuclei to nuclei collisions and neutron capture. While it’s possible that during a supernova for say two Calcium nuclei to fuse and make Zirconium, this is quite unlikely. It’s far more likely that a nucleus of say iron would capture a neutron. Can you say why the latter is more likely than the former?

6) These neutron captures and subsequent beta decays (in which a neutron decays into a proton and spits out a high speed electron) can continue to synthesize elements up to Bismuth-209! The neutron flux in a supernova can be $10^{22}/\text{cm}^2/\text{s}$! These neutron reactions come in many, many varieties and involve too many possibilities of radioactive decay to list, but let it suffice to say that these energetic reactions take place in stupendous amounts and speeds during a supernova explosion.