Chapter 14 EP (1-3, 5-9)

① $f = 440Hz$

$T = 1/f$

$T = 1/440 = \boxed{2.3 \times 10^{-3} s}$

② 10cm to 60cm

10 cycles in 33s

a) Period $= \frac{33s}{10} = \boxed{3.3sec}$

b) Frequency $= \frac{10 cycles}{33 sec.}$ $\boxed{.30 Hz}$

c) Ang. Freq. $= 2\pi f = \boxed{1.9 rad/s}$

d) Amplitude $\frac{60cm - 10cm}{2} = \boxed{25cm}$

e) max speed $= \omega A$

$= \boxed{48 cm/s}$

③ $t_0 = 0 \quad v_0 = 0$

$T = 2.0s \qquad \omega = \frac{2\pi}{T} = \pi$

$v_{max} = 40cm/s$

a) Amplitude $= \frac{v_{max}}{\omega}$

$\frac{40cm/s}{\pi} = 12.7$

$\boxed{13cm}$

b) $x(t) = A \cos(\omega t + \phi_0)$

$x(t) = 12.7324\,cm(\cos \pi t + 0)$

$x(.25s) = \boxed{9.0cm}$

⑤ a) Amplitude is $\boxed{20cm}$

b) looks like it repeats over 4s

so $f = \boxed{.25Hz}$ $\qquad \omega = 2\pi f = \pi/2$

c) $x(t) = A \cos(\omega t + \phi_0)$

$x(t) = 20cm \cos(\pi/2\, t + \phi_0)$

at $t = 0$ $x(t) = 10cm$ - look at graph

so

$10cm = 20cm \cos(0 + \phi_0)$

which means $\cos \phi_0 = .5$

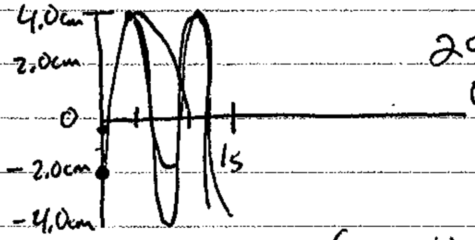
so $\phi_0 = \pi/3$ or $-\pi/3$

since its increasing in x after 0 seconds

must be $\boxed{\phi_0 = -\pi/3 \text{ rad}}$

⑥ $A = 4.0cm \qquad \phi_0 = \frac{2}{3}\pi$ rad.

$f = 2.0Hz \qquad \omega = 2\pi f = 4\pi$



2 cycles over by ≈ 1sec

$x(t) = A \cos(\omega t + \phi_0)$

$x(t) = 4.0cm((4\pi t + \frac{2}{3}\pi)$

at $t = 0$ $x(t) = -2$ cm

$4.0cm = 4.0cm \cos(4\pi t + \frac{2}{3}\pi)$

$4\pi t + \frac{2}{3}\pi = 0$

$t = -1.\overline{6}$ sec

repeats every 2π rad

$4\pi t + \frac{2}{3}\pi = 2\pi$

$t = .33$ sec

⑦ $A = 8.0\text{cm}$

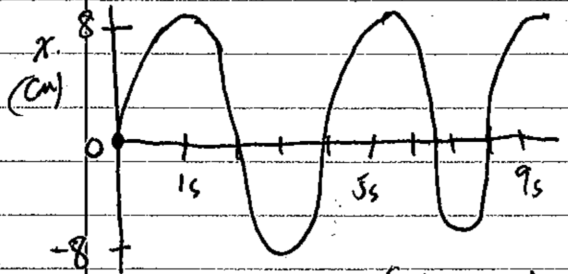
$f = .25\text{Hz} \quad \omega = 2\pi f = \pi/2$

$\phi_0 = -\pi/2$

$x(t) = 8.0\text{cm} \cos\left(\pi/2\, t + -\pi/2\right)$

$x(0) = 8.0\text{cm} \cos\left(-\pi/2\right)$

$x(0) = 0$



$8.0\text{cm} = 8.0\text{cm} \cos\left(\pi/2\, t - \pi/2\right)$

$\cos\left(\pi/2\, t - \pi/2\right) = 1$

$\pi/2\, t - \pi/2 = 0$ or 2π or 4π etc.

$t = 1s$ or $5s$ or $9s$

($T = 4s$ since $f = .25\text{Hz}$)

⑧ $A = 4.0\text{cm}$

$f = 4.0\text{Hz} \quad \omega = 2\pi f = 8\pi\,\text{rad/s}$

at $t = 0$ $x(0) = 0$ going right (+)

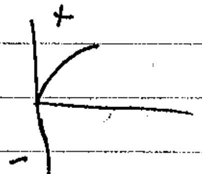
$x(t) = A\cos(\omega t + \phi_0)$

$$\boxed{x(t) = 4.0\text{cm} \cos\left(8\pi\,\text{rad/s}\; t + -\pi/2\,\text{rad}\right)}$$

at $x(0) = 0$

so $\cos(8\pi t + \phi_0) = \cos(\phi_0) = 0$

this happens at $\phi_0 = \pi/2$ or $3\pi/2$ or $-\pi/2$, etc.



$-\pi/2$ so cosine will start out increasing with time

⑨ $A = 8.0\text{cm}$

$f = .50\text{Hz}$

at $t = 0$: $-v_{max}$

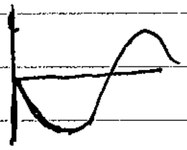
↳ v_{max} at equilibrium position (no velocity at amplitude)

if neg. vel. then moving left...

$\omega = 2\pi f = \pi\,\text{rad/s}$

$x(t) = A\cos(\omega t + \phi_0)$

$$\boxed{x(t) = 8.0\text{cm} \cos\left(\pi\,\text{rad/s}\; t + \pi/2\,\text{rad}\right)}$$



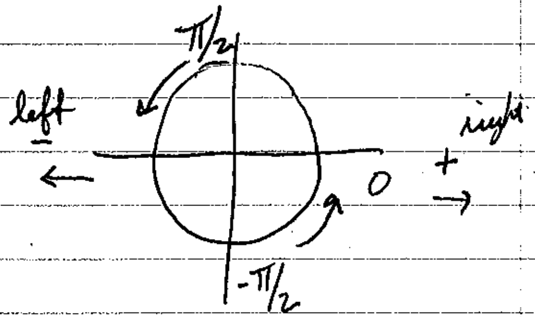
$x(0) = 0$ so

$\cos(\pi(0) + \phi_0) = 0$

so $\cos(\phi_0) = 0$

which happens at $\pi/2$ & $-\pi/2$

Since leftward $\pi/2$



Chap 14 EP (11-15)

⑪ $T_0 = 2s$ $T = 2\pi\sqrt{\frac{m}{k}}$

a) $2m \sim \sqrt{2}T$

$\sqrt{2} \cdot 2s = \boxed{2.8s}$

b) $\frac{1}{2}m \sim \sqrt{\frac{1}{2}}\,T$

$= \boxed{1.41 sec}$

c) $2A$ - $\boxed{\text{no change in } T}$

d) $2k \sim \sqrt{\frac{1}{2}}\,T$

$= \boxed{1.41 sec}$

⑫ $m = .200kg$

$A = 10cm$

$f = \frac{10}{12} = .83\ Hz$

$T = 1.2\ sec$

$T = 2\pi\sqrt{\frac{m}{k}}$

$1.2 = 2\pi\sqrt{\frac{.2}{k}}$

$1.44 = 4\pi^2\left(\frac{.2}{k}\right)$

$\boxed{k = 5.5\ N/m}$

⑬ $m = .200kg$

$f = 2.0Hz$

at $t = 0s \sim x_0 = 5.0cm$

& $v_x = -30cm/s$

13 cont.

a) $T = \frac{1}{f} = \frac{1}{2} = \boxed{.5s}$

b) $\omega = 2\pi f = \boxed{4\pi\ rad/s}$

$\omega = \sqrt{\frac{k}{m}}$

c) $A = ?$

$\frac{1}{2}kA^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2$ $k = m\omega^2$

$m\omega^2A^2 = m\omega^2x_0^2 + mv_0^2$

$(.2)(4\pi)^2(A^2) = (.2)(4\pi)^2(5cm)^2 + (.2)(30cm/s)^2$

$\boxed{A = 5.54cm}$

d) $\phi_0 = ?$

$A\cos\phi_0 = x_0 = 5.0cm$

$\phi_0 = \cos^{-1}\left(\frac{5.0cm}{5.54cm}\right) = \boxed{0.45\ rad}$

e) $v_{max} = \omega A = \boxed{70cm/s}$

f) $a_{max} = \omega^2 A = \boxed{8.8m/s^2}$

g) $E = \frac{1}{2}mv_{max}^2 = \boxed{0.049J}$

h) $t = 0.40s$ $x = ?$

$x(t) = A\cos(\omega t + \phi_0)$

$= 5.54cm\cos(4\pi t + .45)$

$x(.4) = \boxed{3.8cm}$

⑭ $m = .050\,\text{kg}$

$x(t) = (2.0\,\text{cm})\cos\left(10t - \pi/4\right)$

$\frac{dx}{dt} = v(t) = -20\,\text{cm/s}\,\sin\left(10t - \pi/4\right)$

a) $A = 2.0\,\text{cm}$

b) $T = ?$

$10 = \omega = \frac{2\pi}{T}$ so $\boxed{T = .63\,\text{s}}$

c) $k = ?$

$\omega = \sqrt{\frac{k}{m}}$ so $\boxed{k = 5.0\,\text{N/m}}$

d) $\phi_0 = \boxed{-\pi/4\ \text{rad}}$

e) at $t = 0\,\text{sec}$

$\boxed{x(0) = 1.41\,\text{cm}}$

$v(t) = -20\,\text{cm/s}\,\sin\left(10t - \pi/4\right)$

$\boxed{v(0) = 14.1\,\text{cm/s}}$

f) $v_{max} = A\omega = \boxed{20\,\text{cm/s}}$

g) $E = \frac{1}{2}kA^2 = \boxed{1.00\times10^{-3}\,\text{J}}$

h) $t = 0.41\,\text{sec}$

$\boxed{v = 1.46\,\text{cm/s}}$

⑮ $m = 1.0\,\text{kg}$

$k = 16\,\text{N/m}$

$v_0 = v_{max} = 40\,\text{cm/s}$

a) $\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 + \cancelto{0}{\frac{1}{2}kx_0^2}$

$kA^2 = mv_0^2$

$(16\,\text{N/m})$[illegible]$A^2 = (1)(.4\,\text{m/s})^2$

$A = .10\,\text{m} = \boxed{10\,\text{cm}}$

b) at $x = \frac{1}{2}A$ $v = ?$

$\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$

$\cancel{\frac{1}{2}}(16\,\text{N/m})(.10\,\text{m})^2 = \cancel{\frac{1}{2}}(1\,\text{kg})(v^2) + \cancel{\frac{1}{2}}(16\,\text{N/m})(.05\,\text{m})^2$

$.16 = v^2 + .04$

$.12 = v^2$

$\boxed{v = 0.346\,\text{m/s}}$

Chap 14 EP (16-21, 23, 25)

⑯ $m = .5kg$

$\Delta x = .20m$

$F = -kx$

$(.5)(9.8) = k(.2m)$

a) $k = 24.5\ N/m$

b) $\omega = \sqrt{\frac{k}{m}}$ $T = {2\pi}/{\omega}$

$\omega = \sqrt{\frac{24.5}{.5}} = 7$ $\frac{2\pi}{7} = T$

$\boxed{T = .90sec}$

c) $v_{MAX} = A\omega$

$= (.10m)(7)$

$= \boxed{.70m/s}$

as it passes equilibrium

⑰ $k = \frac{mg}{.02}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{mg}{.02}}{m}}$

$= 22.136\ rad/s$

$\frac{\omega}{2\pi} = f = \boxed{3.5 HZ}$

⑱ $k = 15N/m$

$A = .06m$

$\frac{30}{20} = f = 1.5Hz$

$\omega = 2\pi f = 2\pi(1.5)$

$= 3\pi\ rad/s$

$\omega = \sqrt{\frac{k}{m}}$

$3\pi = \sqrt{\frac{15}{m}}$

$9\pi^2 = \frac{15}{M}$

$\boxed{m = .169kg}$

$v_{MAX} = \omega A$

$= (3\pi)(.06)$

$\boxed{= .57m/s}$

⑲ pendulum

$T = 4.0s$ $T = 2\pi\sqrt{L_0/g}$

a) $\boxed{T = 4.0s}$

b) $2L_0 \rightarrow$ $4.0s\sqrt{2} = \boxed{5.7s}$

c) $\frac{1}{2}L_0 \rightarrow$ $4.0s\sqrt{\frac{1}{2}} = \boxed{2.8s}$

d) Assuming A still small $\boxed{T = 4.0s}$

⑳ $\theta(t) = (.10rad)(\cos(5t + \pi))$

a) $A = \boxed{0.10rad}$

b) $f = \frac{\omega}{2\pi} = \frac{5}{2\pi} = \boxed{.796Hz}$

c) $\phi_0 = \pi$ rad.

d) $T = 1/f = 1.2566sec$

$T = 2\pi\sqrt{L/g}$

$\boxed{L = .392m}$

e) $\theta(0) = (.10rad)(\cos\pi)$

$= \boxed{-.10rad}$

f) $\theta(2.0s) = .10rad(\cos(10 + \pi))$

$= \boxed{0.084rad}$

㉑ Assume $\theta \approx \sin\theta$

& SHM

$T = 2\pi\sqrt{\frac{L}{g}}$ $\frac{12s}{10} = 2\pi\sqrt{\frac{L}{9.8}}$

$\boxed{L = .357m}$

(23) $T = 2\pi\sqrt{L/g}$

$= 2\pi\sqrt{\frac{2}{9.8}}$

$T = 2.8384538\,\text{sec}$

$= 2\pi\sqrt{\frac{L}{1.62\,m/s^2}}$

$G\frac{M_M}{r_M^2} = g_{moon}$

$= 1.62$

$\boxed{L_{moon} = 0.33\,m}$

(25) $\omega = 2\pi f = \sqrt{\frac{Mgl}{I}}$

but must find mass

$F = -kx$

$mg = k\Delta x$

$\cancel{mg}\; m = \frac{k\Delta x}{g}$

$m = \frac{(360\,N/m)(.03)}{9.8}$

$m = 1.10\,kg$

$T = .90\,\text{sec} \approx f = 1.\bar{1}\ Hz$

$2\pi(1.\bar{1}) = \sqrt{\frac{(1.10)(9.8)(.14)}{I}}$

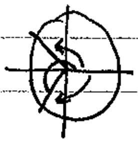
$\boxed{I = 3.1 \times 10^{-2}\ kg \cdot m^2}$

Chap14 EP (31, 32, 41, 58)

(31) $-\frac{1}{2}A = A\cos\phi_0$

$\cos\phi_0 = -\frac{1}{2}$

ϕ_0 is ~~$3\pi/4$~~ or ~~$+5\pi/4$~~



$2\pi/3$ or $5\pi/3$

$-2\pi/3$

getting more neg. so

$\boxed{\phi_0 = 2\pi/3 \text{ rad}}$

$T = 4s$

b) $x(t) = 10\text{cm} \cos(\pi/2\, t + 2\pi/3)$

$\omega = 2\pi/T \quad \omega = 2\pi/4 = \pi/2$

$\frac{dx}{dt} = \left(-\frac{10\pi}{2}\text{cm}\right)\sin(\pi/2\, t + 2\pi/3)$

$\boxed{v(0) = -13.6 \text{ cm/s}}$

c) $v_{max} = \omega A = +5\pi$

$\boxed{= 15.7 \text{cm/s}}$

(32) $T = 12\text{sec}$

$\omega = 2\pi/T = \pi/6 \text{ rad/s}$

$v_{max} = 60\text{cm/s} = \omega A$

$= 60\text{cm/s} = \pi/6\,(A)$

a) $\boxed{A = 115 \text{ cm}}$

b) at $t = 0$

$v(t) = -\omega A \sin\phi_0$

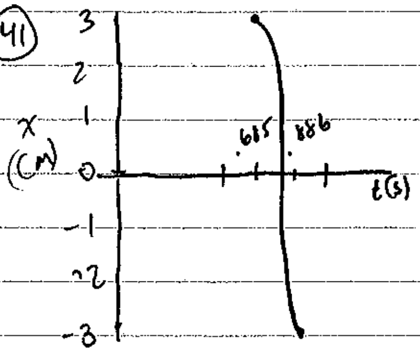
$-30\text{cm/s} = -(\pi/6)(115)\sin\phi_0$

$\boxed{\phi_0 = 5\pi/6} = 2.62 \text{ rad.}$

c) $x(t) = 115\text{cm}(\cos(\pi/6\, t + 5\pi/6))$

$\boxed{x(0) = -100\text{cm}}$

(41)



$x(t) = A\cos(\omega t + \phi_0)$ since released from rest

$\phi_0 = 0$ so $x(0) = A$

$x(t) = A\cos(\omega t)$

$3 = A\cos(\omega(.685))$ $\quad -3 = A\cos[\omega(.886)]$

$3 = -A\cos[(\omega .886)]$

$\cancel{A}\cos(.685\omega) = -\cancel{A}\cos(.886\omega)$

$\cos(.685\omega) = -\cos(.886\omega)$ solver...

or better... cosine flips sign at intervals of π radians so

$.685\omega = \pi - .886\omega$ ...

$\cos(.685\omega) = \cos(\pi - .886\omega)$

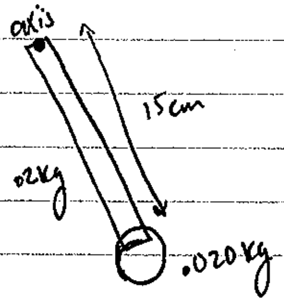
$\boxed{\omega = 2.00 \text{rad/s}}$

b) $x(t) = A\cos(\omega t) \leftarrow .685\text{sec}$

$3\text{cm} = A\cos(2t)$

$\boxed{A = 15 \text{cm}}$

(58)



$$I_{rod+ball} = I_{rod} + I_{ball}$$

$$\tfrac{1}{3} m_{rod} L^2 + m_{Ball} L^2$$

$$= \tfrac{1}{3} (.2)(.15)^2 + (.02)(.15)^2 = 1.95 \times 10^{-3}\ kg \cdot m^2$$

$$y_{cm} = \frac{\Sigma m_i y_i}{M_{total}} = \frac{(.2)(.15/2) + (.02)(.15)}{(.2)+(.02)} = \quad 8.18 \times 10^{-2} m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgl}{I}} = \frac{1}{2\pi}\sqrt{\frac{(.220)(9.8)(8.18E^{-2})}{1.95E^{-3}}} = 1.51\,Hz$$

(l = distance from y_{cm} to pivot)

$$T = 1/f = \frac{1}{1.51} = \boxed{.66s}$$