Chapter 13 CQ (3-6)

(3) a) $F_1/F_2 = 1/2$ $\quad F = G\frac{M_1 M_2}{r^2}$

b) $a_1/a_2 = 1/1$ $\quad F = ma$

(4) Can occur at any distance if astronauts in orbit (assuming they are in free-fall they will be "apparently" weightless - from another perspective they can never be far enough such that the Earth no longer pulls on them)

(5) No. It's in orbit travelling at same velocity as shuttle.

(6) mass 2X $\sim g$ 2X
radius 2X $\sim g$ $\frac{1}{4}$X
so $\frac{1}{2}g = \frac{1}{2}(20\,m/s^2)$
$10\,m/s^2$

Chap 13 EP (4-6, 7, 9, 11)
(See inside back cover for solar system data)

(4) $r = .50\,m$ $\quad F = G\frac{M_1 M_2}{r^2}$

$Pb = 5900\,kg$

$$\frac{F_{Pb} = \cancel{G}\frac{(5900\,kg)\,\cancel{m}}{(.5)^2}}{F_E = \cancel{G}\frac{(5.98E24)\,\cancel{m}}{(6.37E6)^2}} = 1.6\times10^{-7}$$

(5) $$G\frac{(50)(70)}{(1)^2} = 2.3\times10^{-7}\,N$$

(6) $r = 6.37\times10^6\,m + 300{,}000\,m$
$= 6.67\times10^6\,m$

$$F = G\frac{(5.98E24)(1)}{(6.67E6)^2}$$

a) $F = 9.0\,N$

b) in free fall inside shuttle just as shuttle in free fall

(7) $F_W = F_G$

$$\cancel{m}g = G\frac{\cancel{M} M_\odot}{r_\odot^2}$$

$$g = \frac{(6.67E{-}11)(1.99E30)}{(6.96E8)^2}$$

a) $g = 274\,m/s^2$

b) $$g = \frac{(6.67E{-}11)(1.99E30)}{(1.5E11)^2}$$

$= 5.90\times10^{-3}\,m/s^2$

(9) $$g_\oplus = \frac{GM_\oplus}{r_\oplus^2}$$ (you must use $g = 9.83\,m/s^2$ since this matches the mass & radius values.)

$$\begin{pmatrix}9.83\,m/s^2\\ -.0075\,m/s^2\end{pmatrix} = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}\cdot 5.98\times10^{24}\,kg}{(6.37E6 + h)^2}$$

$$9.8225 = \frac{3.98866\,E14}{(6.37E6 + h)^2}$$

lazy so I used TI-89 solver

$h = 2392\,m$

$= 2.4\,km$

(11) $r_z = 5{,}000\,km = 5\times10^6\,m$

$g = 8.0\,m/s^2$

a). $$g = \frac{GM}{r^2}$$

$$8 = \frac{(6.67E{-}11)(M)}{(5E6)^2}$$

$M = 3.0\times10^{24}\,kg$

b) $10{,}000\,km = 1\times10^7\,m$

$r = 5E6 + 1E7$

$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11})(3E24)}{(1.5E7)^2}$$

$g = 0.89\,m/s^2$

Chap13 CQ (7, 8, 10, 11)

(7) It is negative b/c we define $r = \infty$ to be $U_g = 0$ & we are always "under" that position. As objects move toward each other r gets less & lose U_g & gain K.

(8) $$v_{escape} = \sqrt{\frac{2GM}{R}}$$

if twice as dense, the mass is twice as much (radius same)

so

$v_{escape} \times \sqrt{2}$

$10{,}000\,m/s \times \sqrt{2} = 14{,}142\,m/s$

(10) C. 11.9 years - orbital period independent of objects mass - does depend on Sun's mass, but that's not changing

(11) for a circular orbit

$$v = \sqrt{\frac{GM}{r}} \quad \text{and so}$$

curiously (despite drag) the satellite increases speed as it spirals inward (loses U_g & gains K).

EP

Chap 13 (12-19, 22, 24)

(12) $g_{mars} = \frac{G M_{mars}}{r_{mars}^2}$

$\frac{G(6.42E23)}{(3.37E6)^2}$

$= 3.77 \, m/s^2$

so ... same K at launch on both planets - same U_g at top

$\frac{g_E}{g_{mars}} = \frac{9.8}{3.77} = 2.6074\times$ as high

$15 \times 2.6074 = 39m$

(13) You must know how to derive equation for v_{escape}!
(at ∞)

$U_{g_i} + K_i = K_f + U_{g_f}$

$-\frac{G M_J m_o}{R_J} + \frac{1}{2} m_o v_{escp}^2 = 0 + 0$

So

$\frac{1}{2} \cancel{m_o} v_{escp}^2 = + \frac{G M_J \cancel{m_o}}{R_J}$

$v_{escape} = \sqrt{\frac{2 G M_J}{R_J}}$

$= \sqrt{\frac{2(6.67\times10^{-11})(1.90\times10^{27})}{6.99\times10^{7}}}$

$= 60{,}200 \, m/s$

(14) $K_i + U_i = K_f + U_f$ $\quad U_g$ at $\infty = 0$

$\frac{1}{2} m v_i^2 + -\frac{G M_E m}{R_E} = \frac{1}{2} m v_f^2 + 0$

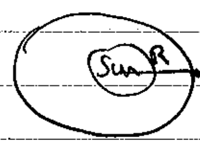
$v_i^2 + -\frac{2 G M_E}{R_E} = v_f^2$

$(15000)^2 - \frac{2(6.67E{-11})(5.98E24)}{(6.37E6)} = v_f^2$

$v_f = 9988$

$\approx 10{,}000 \, m/s$

(15)



$U_{g_i} + K_i = U_{g_f} + K_f$

$-\frac{G M m_o}{r} + \frac{1}{2} m_o v_{esc}^2 = 0 + 0$

$\sqrt{\frac{2GM}{r}} = v_{escape}$

$\sqrt{\frac{2(6.67E{-11})(1.99E30)}{(1.5E11)}} = 42{,}100 \, m/s$

(most of Solar system's mass is located in Sun - if you add mass of other planets - not much effect)

(16) 2x mass

$g = \frac{1}{4} g_{Earth}$

$g = \frac{GM}{r^2}$

$\frac{1}{4} g = \frac{G\,2M}{(\sqrt{8}\,r)^2}$

$2.83\, r_{Earth}$

a) $r = 1.80 \times 10^7 m$

$v_{escape} = \sqrt{\frac{2GM}{r}}$

$= \sqrt{\frac{2(6.67E{-11})(2 \cdot 5.98E24)}{1.8E7}}$

b) $v_{escape} = 9410 m/s$

(17) $T = 5.0$ years (You must know how to derive Kepler's 3rd Law)

For circular orbit

$F = ma$

$F_g = m a_c$

$\frac{G M_{sun} m_{Aster}}{r^2} = M_{aster} \frac{v^2}{r}$

$\frac{G M_{sun}}{r} = v^2 \qquad v = \frac{2\pi r}{T}$

$\frac{G M_{sun}}{r} = \frac{4\pi^2 r^2}{T^2}$

$\frac{G M_{sun} T^2}{4\pi^2} = r^3$

17 cont.) $\frac{(6.67 \times 10^{-11})(1.99E30)}{4\pi^2}(5 \times 31{,}536{,}000)^2 = r^3$

$r = \cancel{2.56 \times 10^{11} m}$ $4.37 \times 10^{11} m$

$\frac{2\pi r}{T} = v = \cancel{10{,}200 m/s}$ $17{,}400 m/s$

(18) $\frac{GM}{4\pi^2} T^2 = r^3$

$\frac{(6.67 \times 10^{-11})(M)}{4\pi^2}(31{,}536{,}000 s)^2 = (1.5E11)^3$

$M = 2.01 \times 10^{30} kg$

(19) $\frac{GM}{4\pi^2} T^2 = r^3$

$\frac{(6.67 \times 10^{-11}) M_{star}}{4\pi^2}(402 \times 24 \times 3600)^2 = (2.2 \times 10^{11})^3$

$M_{star} = 5.22 \times 10^{30} kg$

$g = \frac{GM}{r^2} = \frac{(6.67E{-11})(M)}{(.9E7)^2} = 12.2 m/s^2$

$M_{planet} = 1.48 \times 10^{25} kg$

(22) 350 km = 3.5×10^5 m

$r = 6.37E6 + 3.5E5$

$= 6.72 \times 10^6$ m

$\frac{GM_E}{4\pi^2} T^2 = r^3$

$\frac{(6.67E{-11})(5.98E24)}{4\pi^2} T^2 = (6.72E6)^3$

$T = 5480.5$ sec

$T = 91$ min

$v = \frac{2\pi r}{T} = 7700$ m/s

(24) Mars $T = 24.8$ hours

$M = 6.42 \times 10^{23}$ kg

$r_{Mars} = 3.37 \times 10^6$ m

$\frac{GM}{4\pi^2} T^2 = r^3$

$\frac{(6.67 \times 10^{-11})(6.42E23)}{4\pi^2} (24.8 \times 3600)^2 = r^3$

$r = 2.05 \times 10^7$ m

altitude $= 2.05E7 - 3.37E6$

$= 1.72 \times 10^7$ m

$v = \frac{2\pi r}{T} = \frac{2\pi (2.05 \times 10^7)}{(24.8 \times 3600)}$

$= 1440$ m/s

Chap 13 EP (55, 56, 59, 62)

(55) $m = 1.99E30\,kg$

$r = 10{,}000\,m$

$T = 1.0\,s$

a) $v = \frac{2\pi r}{T} = \frac{2\pi(10{,}000m)}{1.0s}$

$v = 63{,}000\,m/s$

b) $g = \frac{GM}{r^2} = \frac{(6.67E{-11})(1.99E30)}{(10{,}000)^2}$

$g = 1.3 \times 10^{12}\,m/s^2$

c) $\vec{W} = 1.3 \times 10^{12}\,N$

d) altitude = 1km

$r = 10000m + 1000m = 11{,}000m$

$F_g = ma_c$

$\frac{GM_s \cancel{m_o}}{r^2} = \cancel{m_o} r\omega^2$

$\frac{GM_s}{r^3} = \omega^2$

$\frac{(6.67E{-11})(1.99E30)}{(11{,}000)^3} = \omega^2$

$\omega = 9986.2 \frac{rad}{s} \times \frac{60s}{1\,min} \times \frac{1\,rev}{2\pi rad}$

$\omega = 95{,}000$ RPM

e) $\frac{GM_s}{r^3} = \omega^2$ geosynchronous when $\omega = \frac{2\pi rad}{s}$ since $T = 1.0s$

$\frac{GM_s}{\omega^2} = r^3$

$r = 1.5 \times 10^6\,m$

(56) $v = \frac{2\pi r}{T}$ $230{,}000\,m/s = \frac{2\pi(25000 \times 3E8 \times 31{,}536{,}000)}{T}$

$T = 6.4613E15$ sec

2.05×10^8 years

a) 205,000,000 years

b) $5E9 / 2.05E8 = 24$ orbits

c) $\frac{GM_{galaxy}}{4\pi^2} T^2 = r^3$

$M_{galaxy} = \frac{r^3 4\pi^2}{G T^2}$

$= \frac{(25000 \times 3E8 \times 31{,}536{,}000)^3 4\pi^2}{(6.67E{-11})(6.4613E15)^2}$

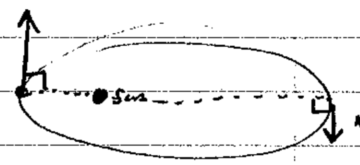
$= 1.87 \times 10^{41}\,kg$

d) $M_{galaxy} \div M_{sun} = 9.43 \times 10^{10}$ stars!

Pluto

(59) at $r = 4.43E9$ km $v = 6.12$ km/s

at $r = 7.30E9$ km $v = ?$



Conserve L, $L = mrv\sin\beta$, $\beta = 90°$

$L_i = L_f$

$\cancel{m_{Pluto}}\, r_i v_i = \cancel{m_{Pluto}}\, r_2 v_2$

$(4.43E9\ \text{km})(6.12\ \text{km/s}) = (7.30E9\ \text{km})(v)$

$v = 3.71$ km/s

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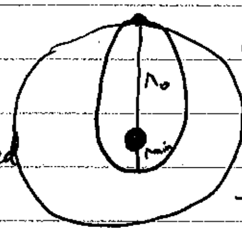
(62) r_0, M, G



new speed = ½ original speed

$F_g = m_s a_c$ circular

$G\dfrac{M\cancel{m_s}}{r_0^{\cancel{2}}} = \cancel{m_s}\dfrac{v^2}{\cancel{r_0}}$

$\sqrt{\dfrac{GM}{r_0}} = v_0$

a) $v_{new} = \frac{1}{2}\sqrt{\dfrac{GM}{r_0}}$

b) $r_{max} = r_0$

Conserve L at either end of orbit

$\cancel{m_0} v_1 r_1 = \cancel{m_0} v_2 r_2$

$v_2 = v_1\left(\dfrac{r_1}{r_2}\right)$

Conserve Energy at either end of orbit

$U_{g_i} + K_i = U_{g_f} + K_f$

$-\dfrac{GM_p\cancel{m_s}}{r_1} + \frac{1}{2}\cancel{m_s}v_1^2 = -\dfrac{GM_p\cancel{m_s}}{r_2} + \frac{1}{2}\cancel{m_s}v_2^2$

$\frac{1}{2}v_1^2 - \dfrac{GM_p}{r_1} = \frac{1}{2}v_1^2\left(\dfrac{r_1}{r_2}\right)^2 - \dfrac{GM_p}{r_2}$

let $r_1 = r_0$ & $v_1 = v_0/2 = \frac{1}{2}\sqrt{\dfrac{GM_p}{r_0}}$

$\frac{1}{2}\left(\frac{1}{2}\sqrt{\dfrac{GM_p}{r_0}}\right)^2 - \dfrac{GM_p}{r_0} = \frac{1}{2}\left(\frac{1}{2}\sqrt{\dfrac{GM_p}{r_0}}\right)^2\left(\dfrac{r_0}{r_2}\right)^2 - \dfrac{GM_p}{r_2}$

$\frac{1}{2}\cdot\frac{1}{4}\dfrac{\cancel{G}M_p}{r_0} - \dfrac{\cancel{G}M_p}{r_0} = \frac{1}{2}\cdot\frac{1}{4}\dfrac{\cancel{G}M_p}{r_0}\left(\dfrac{r_0}{r_2}\right)^2 - \dfrac{\cancel{G}M_p}{r_2}$

$\cancel{\dfrac{1}{8r_0}} - \dfrac{1}{r_0} = \dfrac{1}{8r_0}\dfrac{r_0^2}{r_2^2} - \dfrac{1}{r_2}$

$\dfrac{-7}{8r_0} = \dfrac{r_0}{8r_2^2} - \dfrac{1}{r_2}$

$0 = \dfrac{r_0}{8r_2^2} - \dfrac{1}{r_2} + \dfrac{7}{8r_0}$

$0 = \dfrac{r_0}{8} - r_2 + \dfrac{7}{8r_0}r_2^2$

62. (Cont'd)

$$\left(\frac{7}{8r_0}\right) r_2^2 - r_2 + \frac{r_0}{8} = 0$$

A quadratic eqn.

Use solver

$$\text{Solve}\left(\frac{7}{8r} x^2 - x + \frac{r}{8} = 0, x\right)$$

$x = r$ or $x = .1428571429\, r$

r_2 is r_0 or $r_2 = r_0/7 = r_{min}$

Whew!