Chapter 12 CQ (1-5, 7) EP (2, 4-8)

(1) (a.) closer to larger mass (assuming density same)

(2) $K_R = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(?) m r^2 \omega^2$

$2K_R \approx (\sqrt{2}\, r)^2$

$\sqrt{2}$

(3) Since mass same

$K_R \propto R^2\omega^2$

so $K_a = K_b > K_c$

(4) no. $I \propto m r^2$

no dependence on speed

(5) I larger about end because more mass is farther from axis of rotation

(7) Roll the spheres, the one with less I will accelerate easier → the solid sphere

(2) $\omega_0 = 0$

$\omega_f = 2000 \frac{rot}{min} \times \frac{2\pi\ rad}{rot} \times \frac{1 min}{60s} = 209.44\ rad/s$

$t = 0.50s$

$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{209.44 - 0}{.5} = 418.88\ rad/s^2$

a) $\alpha = 420\ rad/s^2$

$t = .50s = .008\bar{3}\ min$

b) $\theta = \omega_{AVE}\, t$

$= \left(1000 \frac{rot}{min}\right)\left(.008\bar{3}\ min\right) = 8.3$ rotations

(4) $r = 40cm = .40m$

$\omega_0 = 60 \frac{rot}{min} \times \frac{2\pi\ rad \cdot min}{60\ sec \cdot rot} = 6.28\ rad/s$

$\omega_f = 0$

$t = 25sec$

$\alpha = \frac{\Delta\omega}{t} = .2513274123\ rad/s^2$

b) # rev. $\theta = \omega_{AVG}\, t = 30 \frac{rot}{min} \times .41\bar{6}\ min = 12.5$ rev.

a)

$\omega_f^2 = \omega_0^2 + 2\alpha\theta$

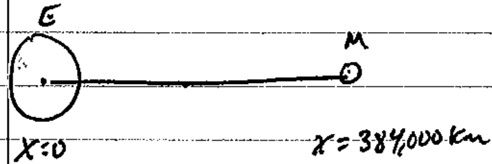
$\alpha = \frac{\omega_f - \omega_0}{t} = -.2513274123\ rad/s^2 = \frac{\omega_f - 6.28...\ rad/s}{10sec}$

$\omega_f = 3.77\ rad/s$

$v_T = r\omega$

$= (.40m)(3.77 rad/s) = 1.5\ m/s$

(5) $\cancel{x_{cm} = \frac{1}{M}\int x\,dm}$



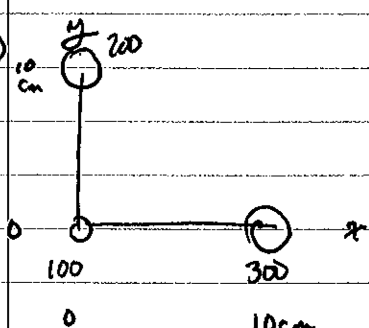
5.98E24 kg　　　7.36 E22 kg

$x_{cm} = \frac{1}{M}\sum m_i x_i$

$x_{cm} = \frac{M_{\oplus}\,0 + (7.36E22)(384000\,km)}{5.98E24 + 7.36E22}$

$x_{cm} = 4670\,Km$
(inside the Earth)

(6)



$x_{cm} = \frac{1}{M}\sum m_i x_i$

$\frac{1}{600g}\left[(200 \cdot 0) + (100 \cdot 0) + (300 \cdot 10cm)\right]$

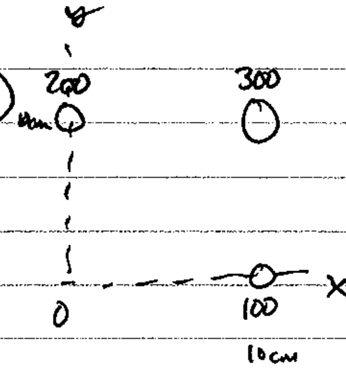
$x_{cm} = 5\,cm$

$y_{cm} = \frac{1}{M}\sum m_i y_i$

$\frac{1}{600}\left[(200)10 + 100(0) + 300(0)\right]$

$y_{cm} = 3.3\,cm$

(7)



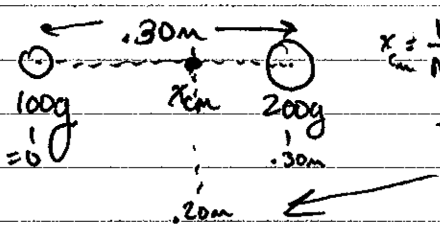
$x_{cm} = \frac{1}{M}\sum m_i x_i = \frac{1}{600}\left[(200 \cdot 0) + 100(10) + 300(10)\right]$

$x_{cm} = 6.7cm$

$y_{cm} = \frac{1}{M}\sum m_i y_i = \frac{1}{600}\left[(200 \cdot 10) + (300 \cdot 10) + (100 \cdot 0\right.$

$y_{cm} = 8.3cm$

(8)



$x_{cm} = \frac{1}{M}\sum m_i x_i$

$\frac{1}{300}\left[(100 \cdot 0) + (200 \cdot .3)\right.$

$\omega = \frac{120\,rot.}{min} \times \frac{2\pi\,rad}{rot} \times \frac{1\,min}{60s} = 12.56637\,rad/s$

$v_T = r\omega$

$= (.20m)(12.56637\,rad/s)$

$= 2.5\,m/s$

Chap 12 EP (9-11, 14-16, 17a, 18a)

(9) I for solid sphere

$I = \frac{2}{5} Mr^2$

$M_\oplus = 5.98 \times 10^{24}$ kg

$r = 6.37 \times 10^6$ m

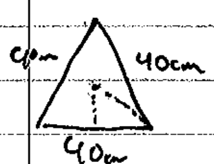
$\omega = \frac{2\pi \text{ rad}}{86{,}400 \text{ sec}} = 7.27220521\text{E-}5$ rad/s

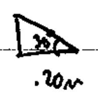
$K_R = \frac{1}{2} I \omega^2$

$= \frac{1}{2}\left(\frac{2}{5}(5.98E24)(6.37E6^2)\right)(7.272205E\text{-}5)^2$

$= 2.57 \times 10^{29}$ J

(10) $I = 3 \times mr^2$





$\cos 30° = \frac{.20\text{m}}{r}$

$r = .2309$m

$I = 3 \times (.200\text{kg})(.2309\text{m})^2$

$= 0.032$ kg m^2

$\omega = \frac{5.0 \text{ rev}}{\text{sec}} = \frac{10\pi \text{ rad}}{\text{s}}$

$K_R = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(.032)(10\pi)^2$

$= 15.8$ J

(11) $m = 0.100$ kg disk $I = \frac{1}{2} Mr^2$

$r = .04$m

$K_R = 0.15$ J

$\omega = ?$

$v = ?$

$v_T = r\omega$

$\omega = v_T / r$

$K_R = \frac{1}{2} I \omega^2$

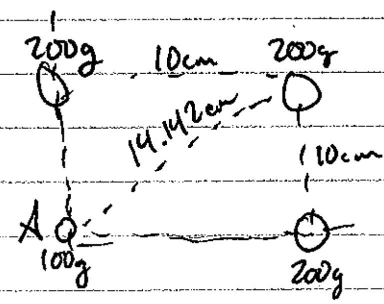
$K_R = \frac{1}{2}\left(\frac{1}{2} M r^2\right)\left(\frac{v_T}{r}\right)^2$

$K_R = \frac{1}{4} M v_T^2$

$.15 = \frac{1}{4}(.1)(v_T)^2$

$v_T = 2.4$ m/s

(14)



$x_{cm} = \frac{1}{M} \Sigma m_i x_i = \frac{1}{700}[(200)10 + 200(10)]$

$x_{cm} = 5.7$cm

$y_{cm} = \frac{1}{M} \Sigma m_i y_i = \frac{1}{700}[200(10) + 200(10)]$

$y_{cm} = 5.7$cm

b) $I = \sum m_i r_i^2$ about point A

$I = (.100\text{kg} \cdot 0^2) + (.2\text{kg} \cdot .1\text{m}^2) + (.2\text{kg} \cdot .14142\text{m}^2) + (.2 \cdot .1^2)$

$I = 0.0080$ kg·m^2

(15) part a) see #14

$x_{cm} = .057m$

$y_{cm} = .057m$

part b) I about diagonal from B to D.

So B & D distance $r = 0$ from axis so only A & C contribute to $\underline{\underline{I}}$.

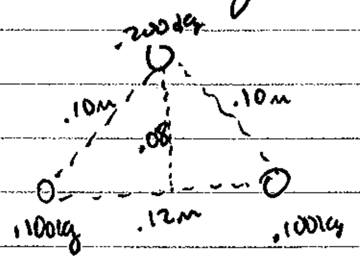
$I = \Sigma M_i r_i^2$

both case $r = \frac{1}{2}(\text{diag.})$
$= .0707m$

(See #14)

$I = (.100kg)(.0707)^2 + (.20kg)(.0707)^2$
$= .0015\ kg \cdot m^2$

(16)



$x_{cm} = \frac{1}{400}\left[100(.12) + 200(.06)\right]$

a) $x_{cm} = .06m$
$y_{cm} = .04m$ by inspection

b) $I_A = (.100kg)(.10m)^2 + (.100kg)(.10m)^2$
$= .0020\ kg \cdot m^2$

c) $I_{BC} = (.200kg)(.08m)^2$
$= .0013\ kg \cdot m^2$

(17a) & b!

$m = 25kg$

$r = .91m$

$I_{hinge} = \frac{1}{3} Ma^2$ see page $\underline{\underline{347}}$

a) $\frac{1}{3}(25)(.91m)^2$
$= 6.9\ kg \cdot m^2$

b) Use parallel axis theorem

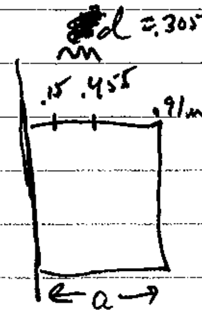
$I = I_{cm} + Md^2$

pg 347

$I = \frac{1}{2} Ma^2 + M(.305)^2$

$= \frac{1}{2}(25)(.91)^2 + 25(.305)^2$

$= 4.1\ kg \cdot m^2$



(18a) & b!

$r = .06m$ $\quad I_{DISK} = \frac{1}{2} mr^2$

$m = .021kg$ $\quad$ pg 347

a) $I = \frac{1}{2} mr^2 = \frac{1}{2}(.021)(.06)^2$
$= 3.8 \times 10^{-5}\ kg \cdot m^2$

b) through edge of disk
Parallel-Axis theorem

$I = I_{cm} + Md^2$

$= \frac{1}{2} mr^2 + Mr^2$

$= \frac{3}{2} Mr^2$

$= \frac{3}{2}(.021)(.06)^2$

$= 1.1 \times 10^{-4}\ kg \cdot m^2$

we are skipping couples (sorry)

Chap 12 EP (19, ~~20~~, 22, 25, 26, 28, 30)

(19) $\tau_{Net} = -30N(.04m) + 20N(.04)$

$= -.40\ N\cdot m$

(22) 20N force produces no torque

$\tau_{Net} = -30N(.10m) + 30N(.05m)(\sin 45^\circ) + 20N(.05m) =$

$= -.94\ N\cdot m$

(25) $I = 2.0\ kg\cdot m^2$

$\alpha = 4.0\ rad/s^2$

$\tau = ?$

$\tau_{net} = I\alpha$

$= (2\ kg\cdot m^2)(4\ rad/s^2)$

$= 8\ N\cdot m$

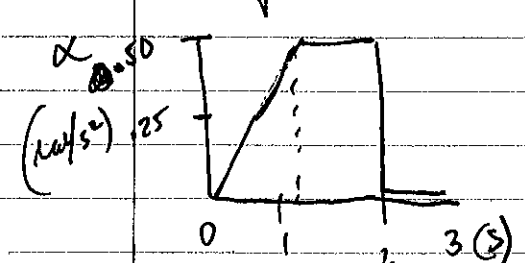
(26) $I = 4.0\ kg\cdot m^2$

$\alpha = \tau / I$

So graph of α vs. t looks same as τ vs. t.

~~except~~ with numerical values of $\tau \div I$

(Can also solve via $\vec{\tau}\cdot t = \Delta \vec{L}$)



area here is $\Delta\omega = \frac{1}{2}(1s)(.5) + (1s)(.5)$

$\Delta\omega = .75\ rad/s = \omega_{3s}$

(28) $m = .200kg$ $r = .10m$

$\omega_0 = 0$

$\omega_f = 1800 \frac{rot}{min} \times \frac{2\pi}{60} = 188.5\ rad/s$

$t = 4.0s$

$\alpha = \frac{188.5}{4} = 47.12389\ rad/s^2$

$I_{Disk} = \frac{1}{2} mr^2$

$\tau = I\alpha$

$= \frac{1}{2}(.2)(.1)^2\ 47.12389$

$\tau = .047\ N\cdot m$

(30) $\tau = I\alpha$

$rF\sin\phi = (I_{Rod} + I_{Rocket})\alpha$

$(.6)(4)\sin 45^\circ = [\frac{1}{3}(.1)(.6)^2 + (.2)(.6)^2]\alpha$

$1.697056275 = .084\alpha$

$\alpha = 20\ rad/s^2$

Chap 12 EP (31-34, 36, 37)

(31) $(.05\,kg)^{(9.8)}(.8m) + (2kg)^{(9.8)}\cdot(.4m)$

$= \cancel{}\ 11.8\,N\text{-}m$

∴ to counter torque from rods own weight & the 500g mass on end

(32) $\Sigma F = 0 = 40 + 60 + -100$

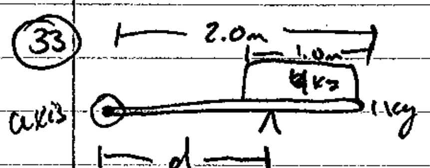
$\Sigma \tau = 0$

let axis be at pt. 100N force applied, then only two torques:

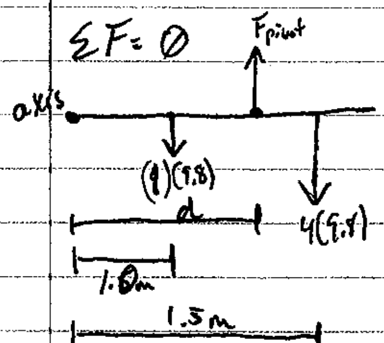
$+60N(1.0m)$ & $-40N(2m)$

but they don't balance so not in equilibrium!

(33)



$\Sigma F = 0$



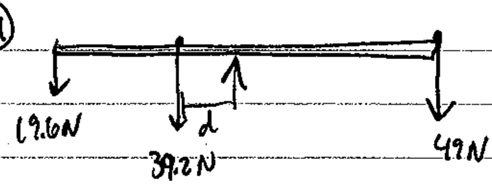
$49N = F_{pivot}$ $(\Sigma F = 0)$

$\Sigma \tau = 0$

$(9.8N)(\cancel{}) + (39.2)(1.5) = (49)(d)$

$d = 1.4m$

(34)



$(19.6)(2) + (39.2)(d) = 49(2)$

$d = 1.5m$

(36) $m = 0.500kg$

$r = .04m$

$v = 1.0m/s$

model as cylinder so $I = \frac{1}{2}Mr^2$

$K = K_T + K_R$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mr^2)(\frac{v^2}{r^2})$

$v = r\omega$

$\omega = \frac{v}{r}$

$\omega^2 = \frac{v^2}{r^2}$

$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$

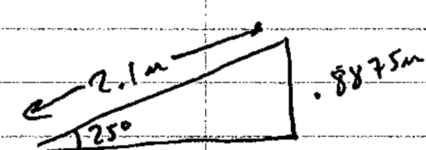
$= \frac{3}{4}mv^2$

$= \frac{3}{4}(.5)(1)^2$

$= \cancel{}\ .38J$

(37) $r = .04m$

$m = 0.400kg$



$\sin 25° = \frac{y}{2.1}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$I_{sphere} = \frac{2}{5}mr^2$

$mgh = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}(\frac{2}{5}mr^2)\omega^2$

$2gh = r^2\omega^2 + \frac{2}{5}r^2\omega^2$

$2gh = \frac{7}{5}r^2\omega^2$

$\frac{10}{7}\frac{gh}{r^2} = \omega^2$

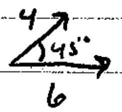
$\omega = 88\ rad/s$

b) $\frac{2}{7}$ ... make sure you got this $\frac{K_R}{K_{total}}$

Chapter 12 EP (38, 39 42, 44, 45, 48, 49, 69-71)

(38) $\vec{A} \times \vec{B} = AB \sin\phi$

a) $(6)(4)\sin 45^\circ$



$= 17$ out of page

b) $\cancel{\vec{A}\times\vec{B}}$ $\vec{C} \times \vec{D} = \emptyset$

since $\phi = 180^\circ$

(39) $(6)(4)\sin 60^\circ$

a) 21 out of page

b) 24 into page

$\vec{C} \times \vec{D}$

(42) look up how to compute

see Example 12.20

use right hand rule

$\hat{x} \times \hat{x} = \emptyset \quad (\hat{i} \times \hat{i})$

$\hat{x} \times \hat{y} = \hat{z} \quad (\hat{i} \times \hat{j})$

$\hat{y} \times \hat{x} = -\hat{z} \quad (\hat{j} \times \hat{i})$

$\hat{y} \times \hat{z} = \hat{x} \quad (\hat{j} \times \hat{k})$

$\hat{z} \times \hat{y} = -\hat{x} \quad (\hat{k} \times \hat{j})$

$\hat{y} \times \hat{y} = \emptyset \quad (\hat{j} \times \hat{j})$

$\hat{z} \times \hat{x} = \hat{y} \quad (\hat{k} \times \hat{i})$

$\hat{x} \times \hat{z} = -\hat{y} \quad (\hat{i} \times \hat{k})$

$\hat{z} \times \hat{z} = \emptyset \quad (\hat{k} \times \hat{k})$

$\vec{A} = 3\hat{i} + \hat{j}$

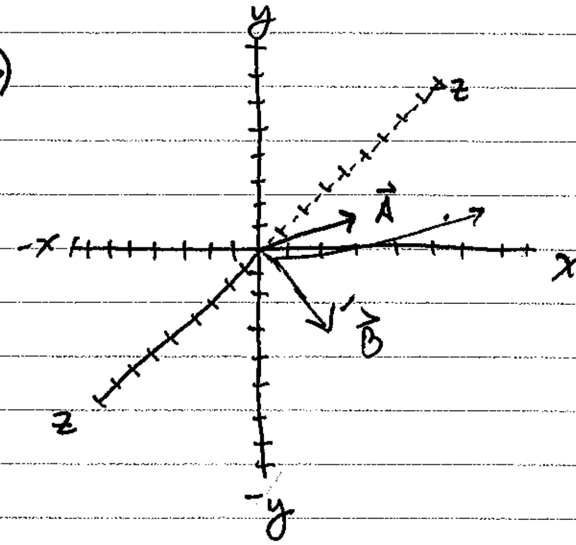
$\vec{B} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$(3\hat{i} + \hat{j})(3\hat{i} - 2\hat{j} + 2\hat{k})$

$9(\emptyset) + -6\hat{k} + {}^-6\hat{j} + {}^-3\hat{k} - 2(\emptyset) + 2\hat{i}$

a) $2\hat{i} - 6\hat{j} - 9\hat{k}$

b)



GL, HF

(44) $\vec{F} = -10\hat{j}$ N $\qquad \vec{r} = (5\hat{i} + 5\hat{j})$ m

$\tau = ?$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = (5\hat{i} + 5\hat{j})(-10\hat{j})$

$= -50\hat{k} - 50(\emptyset)$

$= (-50\hat{k})$ N.m

(45) $\vec{F} = (-10\hat{i} + 10\hat{j})$ N

$\vec{r} = (5\hat{j})$ m

$\tau = \vec{r} \times \vec{F} = (5\hat{j})_m(-10\hat{i} + 10\hat{j})$ N

$= +50\hat{k} + 0$

$50\hat{k}$ N.m

(48) $\vec{L} = I\vec{\omega}$ $\quad \omega = 120 \frac{rot}{min}$

$I = \frac{1}{12} ML^2$

$= 12.56637\ rad/s$

$\vec{L} = \frac{1}{12}(.5)(2)^2 \cdot 12.56637$

$= 2.1\ kg \cdot m^2/s\ \hat{z}$

(49) $\omega = 600 \frac{rot}{min} = 62.83185307\ rad/s$

$m = 2.0 kg$ $\quad I_{Disk} = \frac{1}{2} mr^2$

$r = .02\ m$

$\vec{L} = I\vec{\omega}$

$= \frac{1}{2}(2)(.02)^2\ 62.83$

$= -.025\ kg \cdot m^2/s\ \hat{x}$

(69) $r = 0.75 m$ $\quad I_{Disk} = \frac{1}{2} mr^2$

$m = 250 kg$

$\omega_f = 1200 \frac{rot}{min} = 125.6637\ rad/s$

$\tau = 50 N \cdot m$ $\quad \omega_f = \alpha t$

$\tau = I\alpha$ $\quad \frac{\omega_f}{t} = \alpha$

$50 = \frac{1}{2}(250)(.75)^2 \frac{125.6637}{t}$

a) $t = 177 sec$

b) $K_R = \frac{1}{2} I\omega^2$

$= \frac{1}{2}\left(\frac{1}{2}(250)(.75)^2\right)(125.6637)^2$

$= 555{,}000 J$

c) $P = \frac{\frac{1}{2} K_R}{2 sec} = 139{,}000 W$

d) $\omega_{half\ eng} = ?$

$\frac{1}{2}\left(\frac{1}{2} mr^2\right)\omega^2 = 277{,}591 J$

$\frac{1}{4}(250)(.75)^2 \omega^2 = 277{,}591 J$

$\omega = 88.859\ rad/s$

$\alpha = \frac{\omega_f - \omega_0}{t}$

$= \frac{88.859 - 125.6637}{2}$

$= 18.4\ rad/s^2$

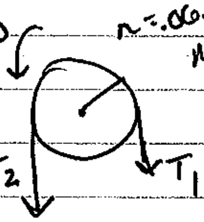
$\tau = I\alpha$

$\tau = \frac{1}{2} mr^2 \alpha$

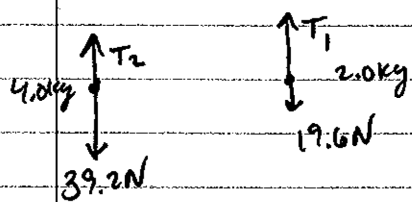
$= \frac{1}{2}(250)(.75)^2\ 18.4$

$\tau = 1300\ N m$

(70) $+\omega$ $r=.06m$ $M_p = 2.0kg$ $\tau_{fric} = -.50N\cdot m$ $I_{Disk} = \frac{1}{2}mr^2$



since pulley not massless or frictionless
$T_1 \neq T_2$!



$a_T = r\alpha$
$\alpha = \frac{a}{r}$

$\Sigma F = ma$ $\Sigma F = ma$ $\Sigma \tau = I\alpha$

~~$T_2 + 39.2N = 4(a)$~~ $T_1 - 19.6 = 2a$ $T_2(.06) - T_1(.06) - .50 = I\alpha$

$39.2 - T_2 = 4a$ $T_2(.06) - T_1(.06) - .50 = \frac{1}{2}(2)(.06)^2\left(\frac{a}{.06}\right)$

$39.2 - \cancel{T_2} = 4a$ $T_2 - T_1 - \frac{.50}{.06} = a$

$\cancel{T_1} - 19.6 = 2a$

$+ \quad \cancel{T_2} - \cancel{T_1} - 8.\bar{3} = a$ $T_2 - T_1 - 8.\bar{3} = a$

$39.2 - 19.6 - 8.\bar{3} = 7a$

$a = 1.60952381\ m/s^2$

Must fall 1m.

$x = \cancel{v_i t}^{\,0} + \frac{1}{2}at^2$

$1 = \frac{1}{2}at^2$

$\sqrt{2/a} = t$

$t = 1.11 sec$

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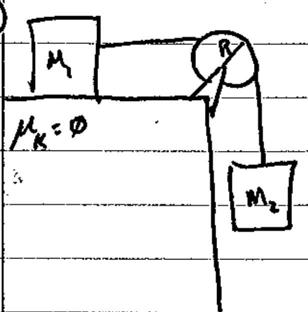
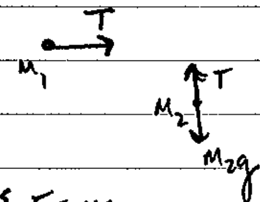


a) assume pulley Massless



$$\Sigma F = ma$$

$$T = M_1 a$$

$$+ \quad M_2 g - T = M_2 a$$

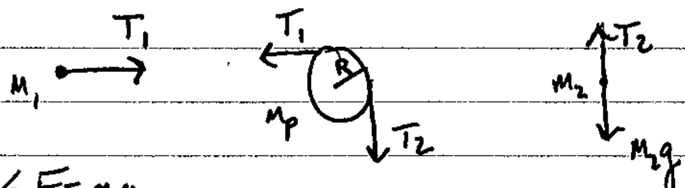
$$M_2 g = (M_1 + M_2) a$$

$$a = \frac{M_2 g}{M_1 + M_2}$$

b) pulley has mass but no friction

$$I_{Disk} = \tfrac{1}{2} mr^2$$

$$\alpha = \frac{a}{r}$$



$$\Sigma F = ma$$

$$T_1 = M_1 a$$

$$\Sigma \tau = I\alpha$$

$$T_2 R - T_1 R = \tfrac{1}{2} m_p R^2 \left(\frac{a}{R}\right)$$

$$T_2 - T_1 = \tfrac{1}{2} m_p a$$

$$\Sigma F = ma$$

$$M_2 g - T_2 = M_2 a$$

$$\cancel{T_1} = M_1 a$$

$$\cancel{T_2} - \cancel{T_1} = \tfrac{1}{2} m_p a$$

$$M_2 g - \cancel{T_2} = M_2 a$$

$$M_2 g = (M_1 + m_2 + \tfrac{1}{2} m_p) a$$

$$\frac{M_2 g}{M_1 + m_2 + \tfrac{1}{2} m_p} = a$$