An Elastic Collision Puzzle:

Imagine a cue ball moving right at 2.0m/s and the eight ball moving left at 3.0m/s. If the collision is elastic and they bounce apart, what are their after collision velocities? (Each has the same mass of 0.40 kg.)

Well, you might be able to predict what will happen from our demonstrations last class, but how can we *calculate* the velocities? Let the two billiard balls be the system, which is a pretty good approximation of an isolated system, so that we can conserve momentum:

Conservation of Momentum	$\sum \vec{p}_{initial} = \sum \vec{p}_{final}$
Total $\vec{p}$ before = total $\vec{p}$ after	$\vec{p}_{c_i} + \vec{p}_{8_i} = \vec{p}_{c_f} + \vec{p}_{8_f}$
$\vec{p} = m\vec{v}$	$m_c v_{i_c} + m_8 v_{i_8} = m_c v_c + m_8 v_8$
$m_c = m_8 = 0.40kg$ (so cancel) +right, -left simplify One Equation, but two unknowns	$(.4)(+2m/s) + (.4)(-3m/s) = (.4)(v_c) + (.4)(v_8)$ $+2m/s - 3m/s = v_c + v_8$ $-1m/s = v_c + v_8$ $-1 = v_c + v_8$
Elastic Collisions can also conserve Kinetic energy since each term has ½ & same mass - cancel all ... simplify	$\sum KE_i = \sum KE_f$ $\cancel{\tfrac{1}{2}}\cancel{m_c} v_{c_i}^2 + \cancel{\tfrac{1}{2}}\cancel{m_8} v_{8_i}^2 = \cancel{\tfrac{1}{2}}\cancel{m_c} v_c^2 + \cancel{\tfrac{1}{2}}\cancel{m_8} v_8^2$ $(+2)^2 + (-3)^2 = v_c^2 + v_8^2$ $4 + 9 = v_c^2 + v_8^2$ $13 = v_c^2 + v_8^2$
-Two equations with two unknowns- solve simultaneously - Solve one for v_c & substitute - algebra...	$-1 = v_c + v_8$ $\quad 13 = v_c^2 + v_8^2$ $v_c = -1 - v_8$ $13 = (-1 - v_8)^2 + v_8^2$ $13 = (1 + v_8 + v_8 + v_8^2) + v_8^2$ $(-13) + 13 = 2v_8^2 + 2v_8 + 1 + (-13)$ $0 = 2v_8^2 + 2v_8 - 12$ — divide by 2 $0 = v_8^2 + v_8 - 6$ — factor or use quadratic formula $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$
* Always get original & final speeds as answers - self check! →	$0 = (v_8 + 3)(v_8 - 2)$ So $v_8 = -3m/s$ → original or $v_8 = 2m/s$ → new velocity!
Substitute $v_8 = 2m/s$ back into one equation to find v_c	$-1 = v_c + v_8$ $-1 = v_c + 2$ $v_c = -3m/s$